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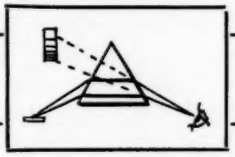

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LORENTZ DOUBLE REFRACTION IN THE REGULAR SYSTEM*

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[Palmer Physical Laboratory, Princeton University, Princeton, New Jersey. Received April 12, 1932]

That the classical crystal optics is simply a first approximation to the actual phenomena, in which the actual inhomogeneities of the electric field due to the finite wave length are neglected, was perhaps first recognized by Lorentz in 1878.[1] Subsequently, in 1882, the same thing was recognized by Gibbs[2] in giving the first electromagnetic theory of optical activity. In 1921 Lorentz[3] returned to the question and showed how the higher approximations lead to a prediction of double refraction even for crystals of the regular system which are optically iostropic in the classical theory.

We shall use the term Lorentz double refraction to mean the double refraction of crystals which is predicted by the higher approximations than those of the classical theory. In this paper we shall give a more complete formulation of the problem than Lorentz has given together with a discussion of the optical properties of the regular or cubic system of crystals. There are many experimental observations showing depar-

* This paper was presented at the Washington meeting of the American Physical Society in April, 1932.

[1] Lorentz, Verh. der Akad. van Wet., Amsterdam (1878).

[2] Gibbs, Amer. J. of Sci. *23*, 460; 1882.

[3] Lorentz, Verh. der Akad. van Wet. *30*, 362; 1921.

tures from the classical crystal optics. These are known as optical anomalies and have usually been attributed to strains or imperfections in the crystal.[4] The existence of these anomalies, some of which are undoubtedly connected with imperfections of particular crystals, makes very difficult anything like an experimental study of the double refraction of crystals of the regular system that is connected with the higher approximations to the theory.

First we sketch the theory of electromagnetic waves in a form which is especially illuminating for problems of crystal optics. Starting from Maxwell's equations

$$\begin{aligned} &\operatorname{div} \boldsymbol{D} = 0 \qquad && \operatorname{div} \boldsymbol{B} = 0 \\ &\operatorname{curl} \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \qquad && \operatorname{curl} \boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} \end{aligned} \tag{1}$$

we make the assumption that $\boldsymbol{E}$ and $\boldsymbol{H}$ depend on position and time through the plane wave form

$$e^{-i\omega(t-\boldsymbol{n}\cdot\boldsymbol{r}/c\eta)} \tag{2}$$

which represents a plane wave travelling in the direction of the unit vector $\boldsymbol{n}$ with phase velocity, $c\eta$ and having the frequency $\omega/2\pi$. Maxwell's equations then lead to the following equations, where now $\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{H}$ mean the constant amplitudes of the plane wave function (2).

$$\begin{aligned} &\boldsymbol{n}\cdot\boldsymbol{D} = 0 \qquad && \boldsymbol{n}\cdot\boldsymbol{B} = 0 \\ &\boldsymbol{n}\times\boldsymbol{E} = \eta B \qquad && \boldsymbol{n}\times\boldsymbol{H} = -\eta\boldsymbol{D} \end{aligned} \tag{3}$$

We assume that

$$\boldsymbol{D} = \boldsymbol{\epsilon}\cdot\boldsymbol{E} \qquad \boldsymbol{B} = \boldsymbol{H} \tag{4}$$

where $\boldsymbol{\epsilon}$ is a tensor. The assumption that the magnetic permeability equals unity corresponds to the usual procedure in theoretical optics but it is doubtful whether it is correct to the order of accuracy which we wish to consider. These equations lead to the following equation to be satisfied by the amplitude $\boldsymbol{D}$,

$$(\boldsymbol{I} - \boldsymbol{nn})\boldsymbol{\epsilon}^{-1}\boldsymbol{D} = \eta^2\boldsymbol{D} \tag{5}$$

which is the fundamental equation of the theory. We observe that it is in the form of a characteristic-value equation so that simple theorems of matrix theory are immediately applicable. In this equation $\boldsymbol{\epsilon}^{-1}$ is the reciprocal tensor to $\boldsymbol{\epsilon}$, while $\boldsymbol{I}$ is the unit tensor or idemfactor of Gibbs.

[4] For a complete account see Brauns, Die optischen Anomalien der Kristalle, Leipzig, 1891.

Suppose we introduce a coordinate system determined by three orthogonal unit vectors, l, m, and n of which the latter coincides with the direction of the wave normal whereas the directions of the former in the wave plane are as yet unspecified. Let us write A for the tensor occurring in (5) so that

$$A = (I - nn)\epsilon^{-1}. \tag{6}$$

Then the components of A in the l, m, n coordinate system can be simply computed in terms of the components in the original i, j, k coordinate system by the usual tensor transformation formulae, involving the direction cosines of l, m, n relative to i, j, k. From the first of equations (3) we know that the component D_n of D must vanish. For this reason (5) reduces to a simple second-order characteristic value equation

$$\begin{pmatrix} A_{ll} & A_{lm} \\ A_{ml} & A_{mm} \end{pmatrix} \begin{pmatrix} D_l \\ D_m \end{pmatrix} = \eta^2 \begin{pmatrix} D_l \\ D_m \end{pmatrix}. \tag{7}$$

It is possible to give a unified presentation of all the optical properties of crystals in terms of the special forms assumed by this matrix. We shall restrict ourselves, for brevity, to the case of non-absorbing crystals for which it is sufficient that the tensor A be Hermitian for all directions of n. In that case the theory of matrices insures that the characteristic values η^2, will have real values (in the cases considered they will also be positive) corresponding to no absorption. Associated with each characteristic value will be a characteristic vector, so that in general we have two indices of refraction and connected therewith two mutually unitary-orthogonal vectors, D, indicating the two states of polarization associated with these velocities of propagation.

If the tensor A is complex and Hermitian then the characteristic vectors will be in general complex, corresponding to the propagation of elliptically polarized light. Under special circumstances the real and imaginary parts of the characteristic vectors may be equal corresponding to circularly polarized light. If the characteristic vectors are real they correspond to two mutually orthogonal directions of plane polarized light.

Thus far we have said nothing about the orientation of the l and m axes in the wave front. For a fixed direction n all the components of A will depend parametrically on a coordinate which fixes this orientation. If we determine this coordinate by the equation

$$\text{Real part of } A_{lm} = 0 \tag{8}$$

then this fixes the orientation of l and m so that these axes are the principal diameters of the ellipses of elliptical polarization or simply

the axes of polarization if the two characteristic vectors correspond to plane polarized light. The cases of circular double refraction and of vanishing double refraction correspond to the case in which (8) is satisfied identically and so this does not determine a particular coordinate system. This is consistent with the fact that in these cases there are no physically distinguished axes in the wave-front. As the proof of these statements is very easy we do not give the details.

Any direction, $\boldsymbol{n}$, in which the double refraction vanishes, i.e., in which the two characteristic values of $\boldsymbol{A}$ are equal, will be called an optical axis.

In case $\boldsymbol{A}$ is real, equation (8) serves to determine the axes in such a way that the tensor is in diagonal form so that the roots for η^2 are simply the values assumed by A_{ll} and A_{mm}. This completes our sketch of the general theory of electromagnetic waves for anisotropic media.

The remainder of the theory of crystal optics falls into two parts, (1) the nature of the general assumption concerning the tensor $\boldsymbol{\epsilon}$ and (2) the investigation of the restrictions placed on these tensors by the symmetry requirements of the special crystal classes. We now consider (1).

Letting P be the electric moment per unit volume we write,

$$P_i = \alpha_{il}E_l + \beta'_{ijl}\frac{\partial E_l}{\partial x_j} + \gamma'_{ijkl}\frac{\partial^2 E_l}{\partial x_j \partial x_k} \tag{9}$$

where a summation over each pair of double indices is implied. The classical theory of crystal optics considers only the first term, i.e., corresponds to vanishing of the tensors β' and γ'. The Gibbs theory of optical activity, later extended by Voigt,[5] corresponds to the retention of the tensor β'. The tensor γ' gives rise to a new kind of double refraction which we have called Lorentz double refraction.

Through the relation,

$$\boldsymbol{D} = \boldsymbol{E} + \boldsymbol{P}$$

and the assumption of plane waves this amounts to a definite assumption for $\boldsymbol{\epsilon}$, namely,

$$\left.\begin{aligned} \epsilon_{il} &= \kappa_{il} + i\beta_{ijl}n_j + \gamma_{ijkl}n_j n_k \\ \kappa_{il} &= \delta_{il} + \alpha_{il} \\ \beta_{ijl} &= \frac{\omega}{c\eta}\beta_{ijl}' \end{aligned}\right\} \tag{10}$$

where

[5] Voigt, Ann. der Phys. *18*, 645; 1905.

and

$$\gamma_{ijkl} = -\frac{\omega^2}{c^2\eta^2}\gamma_{ijkl}'$$

Turning now to 2, we note that for any crystal class (except hemihedry of the triclinic system) restrictions on the components of κ_{il}, β_{ijl} and γ_{ijkl}, in addition to the Hermitian requirement, will be imposed by the crystal symmetry. As an illustration we treat the regular system in detail. We choose coordinate axes along the edges of a cube whose body diagonals are the four three-fold axes of the crystal. The following matrices then give the correspondences of equivalent points for the five classes of the regular system in the sense that if the coordinates of a given point are regarded as a one-column matrix then the one-column matrix which results on multiplying one of the given matrices on the left into the matrix of the point's coordinates, is the matrix of the coordinates of an equivalent point. The matrices may also be regarded as defining certain rotations of the coordinate axes which must leave invariant any tensor which refers to a physical property of the crystal.

Let $(S_{i\sigma})$ be one of the matrices of the symmetry group then the law of tensor transformation gives

$$\bar{\gamma}_{\sigma\tau\mu\nu} = \gamma_{ijkl}S_{i\sigma}S_{j\tau}S_{k\mu}S_{l\nu}$$

as the formula for the tensor components in the new coordinate system in terms of those in the old. We have to restrict γ so that its tensor components are actually left unaltered in value by this process for each S in the group.

I. Tetartohedry (T).

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}\begin{pmatrix}0&-1&0\\0&0&1\\-1&0&0\end{pmatrix}\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}\begin{pmatrix}0&0&-1\\1&0&0\\0&-1&0\end{pmatrix}\begin{pmatrix}0&0&1\\-1&0&0\\0&-1&0\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}0&-1&0\\0&0&-1\\1&0&0\end{pmatrix}\begin{pmatrix}0&1&0\\0&0&-1\\-1&0&0\end{pmatrix}\begin{pmatrix}0&0&-1\\-1&0&0\\0&1&0\end{pmatrix}\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$$

II. Paramorphic hemihedry (T^h).

The group consists of I in addition to those obtained from I by multiplying its elements by

$$\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$$

III. Enantiomorphic hemihedry (O).

The group consists of I in addition to

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

IV. Hemimorphic hemihedry (T^d).

The group consists of I in addition to those obtained by multiplying the new elements of III by

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

V. Holohedry (O^h).

The group consists of the elements of I, II, III and IV.

Applying the symmetry conditions to the tensors, κ, β and γ we have the following results:

1. κ is scalar for all classes, as in classical theory.
2. β vanishes for the classes II, IV and V in which improper rotations occur. For class I the components have the relationships shown in Table

TABLE I

i/jl	11	22	33	23	32	31	13	12	21
1	0	0	0	A	B	0	0	0	0
2	0	0	0	0	0	A	B	0	0
3	0	0	0	0	0	0	0	A	B

I while for class III the symmetry imposes the further requirement that $B = -A$. The Hermitian requirement demands, however, that $B = -A$ in class I so that for each class the tensor β is formally equivalent to the assumption:

$$\boldsymbol{D} = \boldsymbol{E} + iA\boldsymbol{n} \times \boldsymbol{E},$$

which, as is well known, leads through (7) to the property of optical activity or circular double refraction in which the rotatory power is independent of the direction of the wave-normal as in the case of optically active liquids.

3. For classes I and II the form assumed by γ is given in Table II. The way in which this tensor occurs in the optical theory presents only the sums $(B+D)$ and $(C+G)$, and the Hermitian requirement on the

TABLE II

ij/kl	11	22	33	12	21	31	13	23	32
11	A	B	C						
22	C	A	B						
33	B	C	A						
12				D	E				
21				F	G				
31						D	E		
13						F	G		
23								D	E
32								F	G

tensor A in (6) demands further that $(B+D)=(C+G)=H$ so there are only three independent parameters left in the optical effects of this tensor.

For the remaining three classes the symmetry requires a further specialization of the foregoing result in that we must have

$$E = F,\ B = C, \text{ and } D = G.$$

Since the relation of β to optical activity is well known from Voigt's work we shall confine our attention to the case in which β vanishes but γ does not. The general case therefore is that in which κ, A, E, F, and H are neither zero nor equal.

At this point it is interesting to note that for double refraction (except for circular double refraction) any direction which is an axis of greater than two-fold rotational symmetry must be an optical axis. This follows simply from the fact that the two states of polarization are orthogonal in the plane of the wave front. Since each class of the regular system possesses four threefold axes of rotational symmetry each class has at least four optical axes. In addition the axes of the cube are four-fold axes for classes III, IV, and V so for these classes there are at least seven optical axes. The symmetry condition for an optical axis just given is sufficient but not necessary so there may be other optical axes. For example, the optical axes of a biaxial crystal in the usual theory do not fulfill the condition.

For propagation in the direction along one of the coordinate axes we do not need to calculate the reciprocal of ϵ. We use the equation

$$\frac{1}{\eta^2}(I - nn)\boldsymbol{E} = \boldsymbol{\epsilon}\cdot\boldsymbol{E}$$

with the special values $n_1 = n_2 = 0$, $n_3 = 1$ for the direction of propagation relative to the fundamental crystal axes. From this it is readily seen that the two values of the index of refraction are

$$\frac{1}{\eta} = \sqrt{\kappa + F} \text{ and } \sqrt{\kappa + E}$$

corresponding to light linearly polarized along the first and second axes respectively. In accord with the results of the previous paragraph, such a direction is an optical axis for classes III, IV and V for which symmetry demands that $E = F$. Lorentz overlooks the fact that all classes of the regular system do not have three four-fold axes of symmetry and consequently his conclusions, based on electron theory, are only valid for the classes O, T^d and O^h.

To find the double refraction in general we calculate the reciprocal of $\boldsymbol{\epsilon}$ and find the characteristic values and characteristic vectors of (7). For example in the direction $n_1 = n_2 = 1/\sqrt{2}$, $n_3 = 0$, (face diagonal of the fundamental cube) which is the one treated by Lorentz we have, for any class of the system:

$$\epsilon_{ij}^{-1} = a_{ij}/|\epsilon|^3$$

where $|\epsilon|$ is the determinant of the matrix for $\boldsymbol{\epsilon}$ and

$$\begin{aligned}
a_{11} &= [\kappa + \tfrac{1}{2}(E + F)][\kappa + \tfrac{1}{2}(A + E)] \\
a_{22} &= [\kappa + \tfrac{1}{2}(F + A)][\kappa + \tfrac{1}{2}(A + E)] - \tfrac{1}{4}H^2 \\
a_{33} &= [\kappa + \tfrac{1}{2}(F + A)][\kappa + \tfrac{1}{2}(E + F)] \\
a_{13} &= a_{31} = -\tfrac{1}{2}H[\kappa + \tfrac{1}{2}(E + F)] \\
a_{12} &= a_{21} = a_{23} = a_{32} = 0.
\end{aligned}$$

This leads to the following expression for ϵ_{lm}^{-1}

$$\begin{aligned}
|\epsilon|^3\epsilon_{lm}^{-1} = {} & [\kappa + \tfrac{1}{2}(A + E)][\kappa + \tfrac{1}{2}(E + F)]m_1l_1 \\
& - [\kappa + \tfrac{1}{2}(E + F)]\tfrac{1}{2}H(m_1l_3 + m_3l_1) \\
& + \{[\kappa + \tfrac{1}{2}(A + F)][\kappa + \tfrac{1}{2}(A + E)] - \tfrac{1}{4}H^2\}m_2l_2 \\
& + [\kappa + \tfrac{1}{2}(A + F)][\kappa + \tfrac{1}{2}(E + F)]m_3l_3.
\end{aligned}$$

The value of ϵ_{lm}^{-1} vanishes for $l_1 = 1/\sqrt{2}$, $l_2 = 0$, $l_3 = -1/\sqrt{2}$, $m_1 = m_3 = 0, m_2 = 1$, and so these directions determine the planes of polarization. For these directions we have

$$\epsilon_{ll}^{-1} = \frac{\kappa^2 + \frac{1}{2}\kappa[A + \frac{3}{2}(E+F) + \frac{1}{2}H] + \frac{1}{8}(E+F)(E+F+2A+H)}{|\epsilon|^3}$$

$$\epsilon_{mm}^{-1} = \frac{\kappa^2 + \frac{1}{2}\kappa[2A + (E+F)] + \frac{1}{4}[(A+F)(A+E) - F^2]}{|\epsilon|^3}$$

giving the two values of the index of refraction as the reciprocals of

$$\eta_l = \sqrt{\epsilon_{ll}^{-1}} \text{ and } \eta_m = \sqrt{\epsilon_{mm}^{-1}}.$$

The only experimental evidence bearing on this point is that obtained by Lorentz who found a phase difference of between 1/30 and 1/40 of a wave length between the two states of polarization in a one centimeter path with rock salt. The phase difference, Δ, on traversing a distance D is given by

$$\Delta = 2\pi \frac{D}{\lambda}\left(\frac{1}{\eta_l} - \frac{1}{\eta_m}\right) \text{ wave lengths,}$$

where λ is the wave length in air. Assuming that the quantities A, E, and H are of the same order of magnitude and small in comparison with κ Lorentz' experimental result implies a value of about 10^{-9} for these tensor components or a value of the order 10^{-22} cm^2 for a tensor component of γ' in (9).

It would be interesting if more accurate experimental work were done with crystal specimens prepared so as to be as free from internal strains as possible.

INTENSITY, AREA, AND DISTANCE OF VISUAL STIMULUS

By Ellis Freeman
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Area and intensity of visual stimulus[1] are commonly regarded as the determinants of visual functions such as the perception of the minimum separable and of the minimum visible. These two determinants are held to be mutually interchangeable in such a manner that a reduction in the one may be compensated for by an increase in the other. The quantitative relationships between area and intensity have been more recently treated by Piper (1) and Abney (2). Their efforts to equate these variables require a check. Piper selected the expression $K = I\sqrt{A}$, in which K is a constant, I intensity, and A area; Abney selected a still different expression, $K = I \times D$, in which D is the diameter of the stimulus. As both of these equations represent minimal stimulation under dark-adaptation, one or both must be incorrect. Such an equation, if adequate, can be of great importance as a cue to the neural process underlying visual functions.

The present work offers an equation for area and intensity, but secured under more rigidly controlled conditions than those of the others. Secondly, and more important still, it introduces a third, and hitherto entirely neglected factor, namely stimulus-distance, which is equated with area and intensity to give a more comprehensive expression of the determinants of visual functions. Stimulus-distance, as a factor independent of geometrical optics, forced its way to recognition as an important determinant in work recently published (3) (4). It appears that this factor of distance *per se* so influences vision as to give the more distant and proportionately larger stimulus, illuminated at the same intensity as the nearer and proportionately smaller, a markedly finer (smaller) minimum separable and minimum visible. This anomaly cannot be explained away as a pupillary, lenticular, or other artifact, at least as far as known facts go (3).

The aim of the present paper is to express quantitatively, not only the proper relationship between the commonly recognized factors, area

[1] Duration is, of course, another factor, but in regard to the conditions of useful vision it is the least important. Our concern is with continuous stimulation, and since the problem of the effect of duration is understood only in terms of instantaneous stimulation, the latter is of no concern here.

and intensity of stimulus, but also the relationship between these two on the one hand and *stimulus-distance* on the other. Such an expression may give an explanatory cue to the physiological mechanism accountable for the apparent anomaly connected with stimulus-distance, although it has so far failed to yield such a cue. Regardless of this, however, such an equation has the merit, (in giving quantitatively the mutual relationship between these three determinants of vision), of formalizing the status of the one, *distance*, which has been hitherto completely ignored on the assumption that geometrical optics alone governed the relation of stimulus-size to stimulus-distance. In certain practical situations it may be useful in telling more correctly than does geometrical optics the area of stimulus necessary for given intensities at given distances. The concept involved is so far contrary to the principles of geometrical projection, upon which our optics is based, and upon which Snellen Charts are standardized, that it is in order to emphasize its unique nature.

To determine minimum thresholds, as has been done here, the dark-adapted retina must, of course, be used. If, furthermore, the dark-adaptation is maintained throughout the experiment, and only one small retinal patch is stimulated with a very feeble light so that the rest of the retina remains unaffected, a standard starting level of stimulation and a constant and standard background contrast are assured. The data which follow were secured by this method, which involved dark-room observations after a preliminary period of 30 minutes of adaptation, and which employed the apparatus described in (3). In this former paper minimum separable values, corresponding to acuity, were determined, and the graphic presentation was of acuity against the log of intensity. In the present work, in order to preserve the maximum degree of retinal adaptation by stimulation of minimum area, data were secured for the *minimum visible*, which involves only a narrow slit of light against a completely dark background.[2] But the graphic presentation has been rendered wholly logarithmic, and Fig. 1 gives the *log area* against the *log intensity*. The advantage of a wholly logarithmic

[2] The minimum visible and minimum separable, alike, show the anomaly of finer functioning in the large constellation. In the earlier paper acuity values were plotted for the minimum separable, which are the reciprocal of the slit-width for the large constellation and the reciprocal of the slit-width-multiplied-by-10 for the small constellation. If the slit widths for the minimum visible be treated in the same way and plotted against log I, two like curves result as for the minimum separable, except that as slit-widths tend to be appreciably narrower because the dark-adaptation retains its integrity more nearly completely, the reciprocals will be greater.

graph lay in the circumstance that straight lines resulted, which lend themselves more easily to equation-fitting.

The data secured by experiment and the elaboration of these are given in Table 1. Both subjects are the same as those in the first paper. The graphic material in Fig. 1 is based upon the averages of stimulus areas for both subjects and the corresponding intensities in millilam-

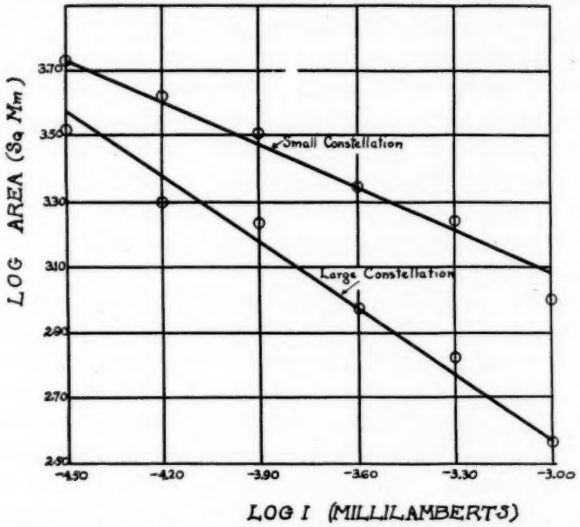

FIG. 1. *Showing the relation between area and intensity of stimulus required for the minimum visible, under dark-adaptation. The circles represent the average of experimental values for both subjects, and as there is no overlapping between constellations, the two constellations may safely be regarded as independent.*

berts. Data for both constellations are given. The averages for these areas have not been plotted, but the logarithms have instead (not given in Table 1). After the logarithms of the average experimental values for stimulus-area from Table 1 had been plotted, it became apparent that each constellation could be represented by a straight line, passing through two values in each case, with the remaining ones falling symmetrically above and below the line.

FITTING EQUATIONS TO THE CURVES

It is obvious that the line for each constellation is represented by the general equation of parabolic type $Y = aX^n$.

Substituting the terms of the data in the above equation, we get

TABLE 1. *Showing, for the minimum visible, under dark-adaptation, the variation of area of stimulus with intensity, for the large and for the small constellation. Stimulus-distance of large constellation, 300 cm, of small 30 cm. N=4.*

Log *I*	Subject 1						Subject 2						Average slit area for both subjects in sq. mm	
	Large constell. Slit 500 mm			Small constell. Slit 50 mm			Large constell. Slit 500 mm			Small constell. Slit 50 mm			Constell.	
	width mm	M.D.	Area sq. mm	width mm	M.D.	Area sq. mm ×100	width mm	M.D.	Area sq. mm	width mm	M.D.	Area sq. mm. ×100	Large	Small
−3.02855	0.5	0.0	250	0.2	0.0	1000	1.0	0.0	500	0.2	0.0	1000	375	1000
−3.32098	0.8	0.0	400	0.3	0.1	1500	1.9	0.3	950	0.4	0.0	2000	675	1750
−3.61341	1.0	0.0	500	0.4	0.1	2000	2.8	0.0	1400	0.5	0.0	2500	950	2250
−3.90584	2.1	0.2	1050	0.5	0.0	2500	4.9	0.3	2450	0.8	0.0	4000	1750	3250
−4.19827	2.6	0.1	1300	0.7	0.0	3500	5.4	0.3	2700	1.0	0.2	5000	2000	4250
−4.49070	4.2	0.4	2100	0.8	0.0	4000	9.1	1.1	4550	1.3	0.1	6500	3325	5250

$A = kI^n$, in which A is area of stimulus, I intensity of stimulus, and k and n constants to be determined.

Expressed in logarithms, it becomes

$$\log A = \log k + n \log I \tag{1}$$

which may now be applied to each line in Fig. 1, to determine the specific form which represents each constellation. To do so, equation (1) in solved for the constant members, log k and n. This requires simply the creation of two simultaneous equations for each constellation with values for log A and log I selected for the two points through which each line has been made to pass.

For the *small constellation*, this results in log $k = 4.86981$ and $n = -.42$. Then substituting these values in (1), we get

$$\log A = 4.86981 - .42 \log I, \tag{2}$$

which expresses the manner in which area of stimulus varies with intensity, at 30 cm stimulus-distance.

For the *large constellation*, following the same procedure, log $k = 4.66373$ and $n = -.69$. Then substituting these values in (1) we get

$$\log A = 4.66373 - .69 \log I, \tag{3}$$

which expresses the manner in which area of stimulus varies with intensity, 300 cm stimulus-distance.

Since equation (2) represents the variation of A and I only at 30 cm stimulus-distance (small constellation); and (3) represents the variation of A and I only at 300 cm stimulus-distance (large constellation), they assert nothing concerning the variation of A and I for stimulus-distances *between* 30 and 300 cm, the range of useful close vision. To secure a final comprehensive equation, it is necessary to establish an internally consistent expression relating the *three* variables, area, intensity, and *stimulus-distance*. This may be done by means of a parametric form, where the parameter D has a range from 30 to 300 cm.

By making the assumption that *between* 30 and 300 cm of stimulus-distance log k and n each varies linearly, the following equation results in the form of a parametric reconciliation of equations (2) and (3):

$$\log A = 4.89261 - .00076D - (.39 + .001D) \log I. \tag{4}$$

Equation (4) is, therefore, the final comprehensive form sought, being internally consistent and representing the three essential variables of vision: area, intensity, and stimulus-distance. If for D we substitute

30, we secure a return to equation (2); if we substitute 300, we secure a return to equation (3); thus demonstrating the internal consistency of the parametric equation (4) with the experimental data.

To make equation (4) suitable for any stimulus-distance between 30 and 300 cm, we need but substitute the desired value for D. Thereupon the relation between A and I may be calculated for the particular stimulus-distance selected.

It has been shown elsewhere (4) that a like anomaly in favor of the large constellation exists for the *light-adapted periphery* of the eye. The results presented in this present paper suggest, consequently, the propriety of applying parametric equation (4), or a similar one, to define the functioning of the entire retinal area, under all degrees of adaptation and all intensities of stimulus.

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ON THE PENETRATION OF DAYLIGHT INTO THE SEA

By E. O. Hulburt
[Naval Research Laboratory. Bellevue, Anacostia, D.C. Received April 1, 1932]

ABSTRACT

During descents into the sea to depths of over 1400 feet Beebe observed that the color of the water illuminated by the sun was a pure blue and measured the candle power of the illumination at various depths down to 800 feet. At 800 feet the spectrum was a narrow band centering at about wave length 520mμ. Using the light absorption coefficients of sea water as measured in the laboratory and the Raman-Einstein-Smoluchowski theory of the scattering of light in liquids, it is found that the spectrum of the daylight at 800 feet in the sea has a maximum brightness at about 500mμ; this agrees well enough with the observation.

The calculated candle powers of the undersea daylight illumination were much greater than the observed values. The assumption that the absorption in the sea below 200 feet was the same as the laboratory measured absorption and in the surface layers down to 200 feet was about 5 times the laboratory value led to good agreement between the observed and calculated candlepowers at all depths. The surface layer absorption would be accounted for if 25 millionths of the volume of the surface water was occupied by opaque material.

Introduction

During June, 1930, in the open sea about ten miles south of Bermuda William Beebe performed a unique journey of exploration.[1] He made a number of descents into the sea in a hollow metal sphere and in his longest dive reached a depth of 1426 feet below the surface. The sea was about one and a half miles deep at the place. The metal sphere had a quartz window in its side and looking out of the window during the midday hours of a sunny day he observed that the light in the water was a brilliant bluish green at a depth of 50 feet, that at 100 feet there was a "slight twilighting and chilling of the blue" and that the blue color became purer and weaker as he descended to greater depths. Describing the appearance at 500 feet Beebe wrote, "I began to see what a strange illumination the water at this depth possessed. In fact, the most amazing thing, that which endured most vividly in our memory was the color." Between 1250 and 1300 feet "not . . . an organism was seen; it was fifty feet of terrible emptiness, with the blue mostly of some wholly new color term—a term quite absent from any human language. If was probably sheer imagination but the characteristic most vivid was its transparency. As I looked out I never thought of feet or yards of visibility but of the hundreds of miles of this color

[1] Beebe, Bull. New York Zoological Society, *33*, 201; 1930.

stretching over so much of the world." At 1400 feet "the outside world . . . seemed born of a single vibration—blue, blue, forever and forever blue."

BEEBE'S OBSERVATIONS

During various dives in the open ocean Beebe found that there was sufficient daylight to depths of 600 feet to enable him to "see and recognize—and even accurately identify—fish in many cases down to the very species." On the tenth dive, down to a depth of 800 feet, he made spectroscopic and intensity observations of the light as he looked in a

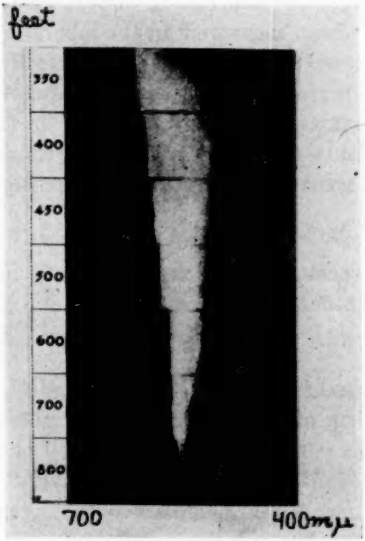

FIG. 1. *The spectrum of daylight in the sea at various depths as observed by Beebe.*

horizontal direction out of the quartz window. By means of a small Zeiss visual spectroscope he observed that each end of the visible spectrum diminished in intensity as the depth increased until at 800 feet the spectrum consisted of only a narrow band in the blue at the wave length about 520$\mu\mu$ which faded out abruptly on the long wave side and not quite so abruptly on the short wave side. The spectra at various depths are shown in Fig. 1, which is a photograph of a portion of Beebe's figure on page 227, reference 1, the figure being the reproduction of a painting. Beebe measured the brightness of the light with a

"Cinophot," a small device used for determining photographic exposures. The Cinophot readings were later translated into brightness units, i.e., candles ft^{-2}, by a comparison with a calibrated incandescent lamp. The observed values of the brightness, denoted by B_h candles ft^{-2}, are given in column 2, Table 2. The value of B_h at 500 ft was 0.02 candles ft^{-2}, and Beebe remarked "as far as my eye could record, there was no further change from 500 feet to the extreme depth of this particular dive, of 800 feet." At 800 ft he could read large print. The temperature during the dive varied from 293° to 297.5° Kelvin.

The absorption of light by samples of pure sea water taken from the Gulf Stream, the Atlantic and Pacific Oceans, and the Caribbean Sea far from land have been measured[2] in the laboratory and it was of interest to see how these measurements applied to Beebe's observations. In the following paragraphs a fairly complete quantitative explanation of Beebe's observations is worked out based on the laboratory light absorption coefficients of sea water and the Raman-Einstein-Smoluchowski[3] density fluctuation theory of the scattering of light by liquids.

THE COLOR OF THE DAYLIGHT IN THE SEA

The absorption coefficient γ for monochromatic light of wave length λ is defined by the usual relation

$$i = i_0 e^{-\gamma z}, \tag{1}$$

where i_0 is the original intensity of the light and i is the intensity after passing through z cm of sea water. The measured values of γ for sea water[2] were 0.0030, 0.00023, 0.000345, 0.00069 and 0.0023 for wave lengths 366, 436, 546, 578 and 612 mμ, respectively. These were plotted as small circles in curve 1, Fig. 2, and a smooth curve was drawn through them. From the curve the values of γ at every 20$\mu\mu$ were read off and are given in column 2, Table 1. It is seen that some of the values in the table depend on a fairly extended interpolation between the observed points on the curve and are therefore open to some uncertainty.

A beam of light passing through a nonfluorescing liquid free from suspended particles loses energy in two ways, by true absorption and by scattering. The energy degradation due to scattering is given by (1) when γ is replaced by the Raman-Einstein-Smoluchowski scattering absorption coefficient α, where

[2] Hulburt, J.O.S.A. and R.S.I. *13*, 553; 1926; *17*, 15; 1928.

[3] Raman, Proc. Roy. Soc. *101*, 64; 1922; *Molecular Diffraction of Light*, Calcutta University Press (1922).

$$\alpha = \frac{8\pi^3}{27}\frac{\sigma RT}{N\lambda^4}(\mu^2 - 1)^2(\mu^2 + 2)^2. \qquad (2)$$

μ is the refractive index, λ is the wave length of the light, $R = 8.315 \times 10^7$ erg deg^{-1} mole^{-1} is the gas constant per gram molecule, $N = 6.06 \times 10^{23}$ is the Avogadro number per gram molecule, $\sigma = 4.95 \times 10^{-11}$ cm^2 dyne^{-1} is the isothermal compressibility of water and T is the absolute temperature which was 295°K. The values of α for pure water calculated from (2) are given in column 3, Table 1. Equation (2) should be multiplied by a correction factor due to the imperfection of polarization, theoretically

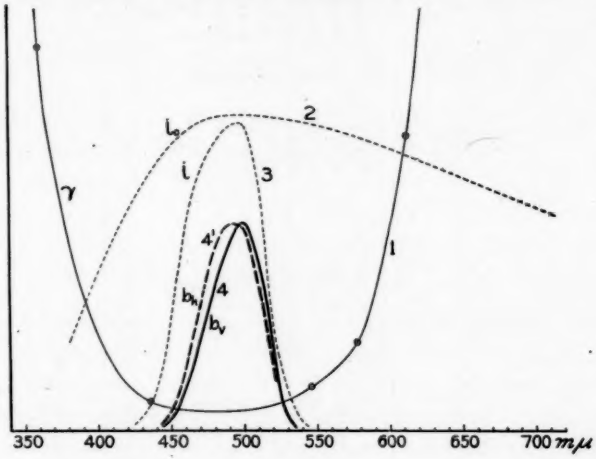

FIG. 2

attributed to the anisotropy of the molecules. Exact values of the correction are not available for water throughout the visible spectrum but it is probably less than 30 percent,[4] and we neglect it. The change in σ and μ with pressure is negligibly small for the pressure increase from the surface to the 800 feet depth. Altogether it seems that no great error is made in taking the values of α of Table 1 to refer to the water of the sea.

The true absorption coefficient is $\gamma - \alpha$ and from the values of Table 1 it is seen that $\gamma - \alpha$ is always greater than 3α. This means that we may to a fair approximation neglect the rescattering of light back into the beam and calculate the passage of light through the ocean using (1) and the values of γ of Table 1.

[4] Krishnan, Phil. Mag. *50*, 697; 1925.

TABLE 1.

1 λ	2 γ	3 α	4 i_0	5 i	6 β/γ	7 $i\beta/\gamma$	8 ψ	9 b_v	10 b_k
400$\mu\mu$	9.32×10^{-4}	6.8×10^{-5}	2.10	0.00×10^{-2}	0.436×10^{-2}	0.00×10^{-3}	0.0115	0.00×10^{-2}	0.00×10^{-4}
420	4.14	5.4	2.87	0.00	0.780	0.00	0.036	0.00	0.00
440	2.30	4.61	3.55	1.32	1.20	0.181	0.087	0.115	0.158
460	1.66	3.9	4.08	7.60	1.40	1.06	0.216	1.64	2.30
480	1.54	3.28	4.24	9.92	1.28	1.27	0.515	5.12	6.54
500	1.61	2.76	4.27	8.50	1.03	0.875	0.847	7.20	7.40
520	1.95	2.25	4.25	3.70	0.69	0.255	0.996	3.68	2.54
540	2.98	1.90	4.19	0.30	0.378	0.01	0.944	0.27	0.09
560	4.14	1.64	4.10	0.00	0.237	0.00	0.735	0.00	0.00
580	7.46	1.42	3.97	0.00	0.114	0.00	0.464	0.00	0.00
600	16.5	1.33	3.83	0.00	0.048	0.00	0.238	0.00	0.00

The values of i_0, the intensity of the light of the sun at the surface of the sea falling vertically through a cloudless but not necessarily hazeless atmosphere,[5] are given in arbitrary units in column 4, Table 1; they are plotted in curve 2, Fig. 1. The sunlight passes down into the sea and, neglecting the loss by reflection at the surface, is reduced in intensity to a value of i at the depth z according to (1); i for $z=800$ feet is given in column 5, Table 1. The light descending vertically through the water is scattered laterally, a fraction βi being scattered per unit volune per unit solid angle in a direction normal to the beam, i.e. in a horizontal direction, where β is given by

$$\beta = \frac{\pi^2}{18}\frac{\sigma RT}{N\lambda^4}(\mu^2 - 1)^2(\mu^2 + 2)^2. \tag{3}$$

From (2) and (3) $\beta = 5.97 \times 10^{-2}\alpha$. The total intensity i_h of the energy passing horizontally is for each wave length

$$\begin{aligned} i_h &= \int_0^\infty \beta i e^{-\gamma x} dx, \\ &= i\beta/\gamma, \end{aligned} \tag{4}$$

where x is the distance measured in the horizontal direction. The values of β/γ are given in column 6, Table 1, and of $i\beta/\gamma$ in column 7.

The brightness or candle power of the light is obtained by multiplying the value of the light energy at each wave length by the sensitivity ψ, or the visibility function, of the eye at that wave length. The visibility curve of the light adapted eye has its maximum at about $550\mu\mu$ and the visibility curve of the dark adapted eye is the same but shifted about $48\mu\mu$ toward the blue.[6] We assume that the average adaptation of the eye to the under sea illumination down to 800 feet was such that the maximum of the visibility curve was at $520\mu\mu$. Accordingly the visibility data[7] for the light adapted eye have been shifted so that the maximum is at $520\mu\mu$. This yielded the values of ψ given in column 8, Table 1. Denoting the brightness at each wave length of the light passing in a vertical and in a horizontal direction, respectively, by b_v and b_h we have

[5] Abbott, Fowle, and Aldrich, Annals of the Astrophysical Observatory of the Smithsonian Institution, *3*, 197; 1913.

[6] Hecht and Williams, J. of Gen. Physiology of the Rockefeller Institute for Medical Research, *5*, 1; 1922–23.

[7] Hyde, Forsythe and Cady, Astrophysical J. *48*, 65; 1918.

$$b_v = \psi i,$$

and

$$b_h = \psi i\beta/\gamma.$$

b_v and b_h for a depth of 800 feet are given in columns 9 and 10 of Table 1, respectively, and are plotted in curves 4 and 4′ Fig. 2, respectively, with maximum ordinates arbitrarily made equal. Shifting the visibility curve $20\mu\mu$ up or down the spectrum shifts curves 4 and 4′ about $10\mu\mu$ up or down the spectrum. The change in the absorption and dispersion of the spectroscope with wave length is small in the spectrum interval occupied by the curves and has a negligible effect on the shape of the curves. Curves 4 and 4′ give, respectively, the spectral distribution of the light seen at 800 feet below the surface looking upward and horizontally. The curves are fairly close together and therefore the color of the light in the two cases is much the same. Curve 4′ is the one perhaps to be compared with Beebe's observations, it is the theoretical representation of the last spectrum of Fig. 1. As far as can be judged it agrees with the observations with, however, a slight discrepancy. The maximum of the curve is at about $490\mu\mu$ whereas the last trace of the spectrum which Beebe saw was at about $520\mu\mu$. Probably neither the accuracy of the data on which the curve is based nor the precision of the observations was sufficiently good to enable one to decide whether the discrepancy is real or not.

From all this the physical causes of the color of the daylight illumination at great depths in the sea may be stated as follows: Two causes are operative, selective absorption and Raman-Einstein selective scattering. If there were no scattering and only selective absorption, the light coming down from above would be blue and there would be no light passing in a horizontal direction. If there were no absorption and only selective scattering, the light coming down from above would be reddish and the light passing in the horizontal direction would be blue; this would be an accentuated counterpart of the color of the sky. When both causes operate, as they actually do, the light coming down from above is blue because the selective absorption predominates over the selective scattering and the light in the horizontal direction is the blue vertical light made slightly more blue by the selective scattering. In saying this we are leaving out of account other possible effects such as fluorescence, scattering or absorption of light by impurities or suspended matter, etc.

THE BRIGHTNESS OF THE DAYLIGHT IN THE SEA

The total brightness or candle power of the light is

$$B_v = \int_0^\infty b_v d\lambda$$

and

$$B_h = \int_0^\infty b_h d\lambda,$$

where B_v and B_h refer, respectively, to the light seen in the vertical and horizontal directions. B_v and B_h were calculated from the areas of the b_v, λ and b_h, λ curves at various depths using 10,000 candles ft^{-2} for the candle power of noon sun light plus sky light at sea level. The values of B_h are given in column 3, Table 2, and are seen to be much greater than the observed values. Likewise, B_v came out too great, for example, being 86 candles ft^{-2} at a depth of 800 feet which would be the illumination of a brilliantly lighted room. Unquestionably the sea near Bermuda was more highly absorbing of light than the water which was used in the measurement of γ.

TABLE 2

Candles per square foot

Observed by Beebe		Calculated		
z	B_h	B_h	B_h'	B_v'
0 feet	—	—	—	10000
50	10.4	44.4	10.5	1060
100	3.37	32.5	3.4	340
200	0.41	17.4	0.41	41
250	0.21	13.0	0.21	21
300	0.15	10.0	0.16	16
350	0.12	7.5	0.116	12
500	0.02	3.5	0.061	6
800	0.02	0.8	0.013	1

There is a further complication which, like all good complications, points the way to an explanation. The logarithms of the calculated values of B_h, column 3, Table 2, were plotted against z in curve 2, Fig. 3. B_h is very closely an exponential function of z, as is shown by the fact that the curve is nearly a straight line. The logarithms of Beebe's observed values of B_h, column 2, Table 2, were plotted against z in

curve 1, Fig. 3, which is made of circles joined by straight full lines. Making some allowance for uncertainties of observation it is seen that the curve is not a straight line, and therefore the observed B_h is not an exponential function of z over its entire length. Curve 1 makes a bend at about $z = 200$ feet and may be regarded roughly as being made of two straight lines of different slopes. This suggests that we assume that the absorption of the Bermuda sea water down to the 250 foot level was greater than the measured values of γ of Table 1 and was such as to give the observed value of B_h at $z = 250$ feet, and that for depths below 250 feet the absorption was the same as γ. The values of B_h, denoted

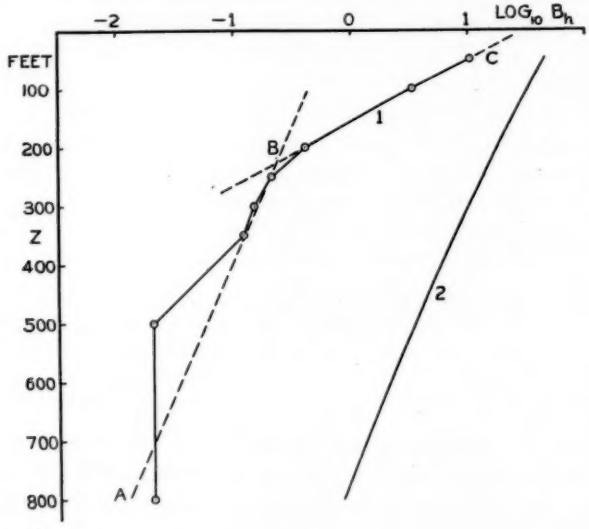

FIG. 3

by B_h', calculated from these assumptions are given in column 4, Table 2, and are also shown by the two straight dotted lines AB and BC, Fig. 3. AB of course has the same slope as curve 2. B_v' corresponding to B_v is in column 5, Table 2. B_h' is seen to agree well with Beebe's values, and the agreement indicates that the density fluctuation scattering theory applied to the deep sea water near Bermuda and that the absorption of the water was the same as that of the laboratory measurements.

Above the 250 foot level the absorption came out 4.8 times the values of γ of Table 1. An absorption of this amount would be equivalent to

that produced by one small fish per cubic meter of size $6 \times 2 \times 2$ cm, or by opaque motes in the water, of animal, vegetable or mineral origin, of area about 0.1 square millimeter and 1 mote per cubic centimeter. If the motes were smaller the number of them would be proportionately larger.

I do not know of course whether the foregoing explanation is correct in all its details. In any event it would be very interesting to have further observations of the intensity and the spectral distribution of the light seen upward and horizontally at many depths in the sea. Observations of the amount of the polarization of the light would also be valuable. From such data one might derive more complete information about the absorption and scattering of light by the water of the sea.

A TRICHROMATIC COLORIMETER

By J. Zernike
[Natuurkundig Laboratorium der N. V. Philips' Gloeilampenfabrieken, Eindhoven, Holland. Received February 15, 1932]

Preliminary Paper

Though the color body is tridimensional, yet owing to its peculiar shape an infinite number of colors is wanted, in order to produce all existing colors. For all practical purposes, however, this number may be reduced to four, namely a red at or beyond 680, a yellowish green at about 535, a bluish green at about 508, and a violet at or beyond 430. If it is desired only to measure pigment colors, the third color at 508 may even be dispensed with, as will be seen from Fig. 1, taken from a paper by K. W. F. Kohlrausch[1]. By the same token the red may lie as far as 620.

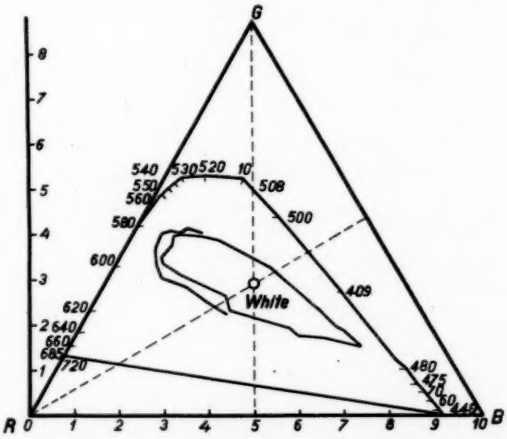

Fig. 1. *Color triangle according to Kohlrausch showing position of pigment colors.*

Several colorimeters, based on the principle just laid down have been constructed in the past, all of which used a source of continuous light, from which three parts in the red, the green and the violet were cut out either by suitable filters or by the use of monochromators.

To the use of filters may be objected: 1st. Owing to the fact that the three colors are not saturated, the field they embrace is smaller. 2nd.

[1] Phys. ZS., *21*, 401; 1920.

It is difficult to reproduce the filters, hence uncertainty exists whether a color measured with one apparatus will give the same result when measured with another. 3rd. The stability of the filters should be very high, in order to ensure identical color matches at different times.

The use of monochromators does away with all these difficulties, but entails another one, namely of obtaining sufficient intensity. Also the price of such a colorimeter is prohibitive.

The advantages of monochromators without any of these drawbacks, are obtained by the apparatus here described, making use of spectral light sources, such as a neon lamp for the red, and two mercury lamps

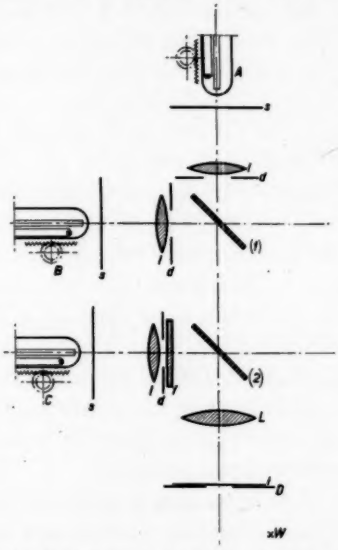

FIG. 2. *Plan view of trichromatic colorimeter with spectral light sources.*

for the green and violet. There remains only the difficulty of ensuring sufficient stability of these lamps.

This requirement may be materially reduced by the following expedient. The intensity of each light source can be varied in two independent ways. First by a variable diaphragm, the aperture of which can be read off. Secondly by moving the light sources to and fro in front of a screen by a rack and pinion device. In making a measurement the diaphragms are opened wide and a standard white is matched by the use of the rack and pinion devices. Next the color which it is desired to measure is put in place of the standard white and is matched by

suitably closing the diaphragms. It is seen that in this way the sources of light need only be constant during the time required for one measurement. Of course if a number of colors are to be measured, the standard white may be checked from time to time. This procedure is entirely comparable to the one used with an analytical balance.

Another difficulty with which apparently all constructors of colorimeters have had to cope with, consists in the adequate mixing of the colors; the requirement being that the variable field should be of exactly the same color and appearance all over. Especially with white and the nearly white colors this is very hard. After many trials a device of half mirrors was had recourse to. A plan view of the apparatus is given in Fig. 2. Even if we suppose the mirrors to be in the ideal condition of reflecting 50% and of transmitting 50%, the intensity of A and B is reduced to 25% and of C to 50%. To get round this point selective mirrors are used. (1) is formed by a thin film of indigo carmine, (2) by one of methyl violet on plate glass. Now if for A and C mercury lamps are used and for B a neon lamp these mirrors also serve the purpose of filters. (1) and (2) nicely filter out the line 435 from A, and (2) filters out the red neon lines 638 and 627. As to the green line 546 to be given by C an additional filter has to be had recourse to; this filter consists of didymium glass carrying a thin film of gelatine stained with picric acid.

The objections raised above against the use of filters do not so forcibly apply here. From time to time the synthetized white should be examined with a (very simple) spectroscope to make sure that only the wave lengths just mentioned are present.

The further optics of the apparatus is easily seen from Fig. 2. s denotes three screens of opal glass, illumined by the lamps, placed in the focuses of lenses l; close behind these are the variable diaphragms d; lens L forms an image of all three screens at i, from which the central portion is cut out by a fixed diaphragm D. The object the color of which is to be measured (fabric, paper etc.) is mounted against D, so that it covers half the opening. It is illumined by a daylight lamp W, the intensity of which can be varied to three distinct different values, so that it gives an illumination at D of 1500, 500 and 150 lux respectively. The darkest black still reflecting 3% in this way any color can be measured with an apparent illumination of 45 to 150 lux. According to Coblentz and Emerson[2] this is the most suitable range of illumination for the purpose.

[2] Bull. Bur. of St. No. 303 (1917).

The usual objection to the type of apparatus described is, that it is not independent of the human eye. It is also objected that they are not easy to handle.[3] As to the first objection it has been found in actual practice, that the settings of different observers do not vary more than the mean error of the settings themselves. This point, however, should be verified on a larger number of observers, especially including both sexes, before a definite conclusion can be laid down. On the other hand it is an advantage that the apparatus actually measures color.[4] The second objection is true enough if one goes by a first impression. A practice of a few days, however, suffices to acquire the necessary routine.

From the foregoing it will be seen that the apparatus characterizes a given color by three numbers, indicating the intensity of three standard wave lengths. These standards are certainly arbitrary, but nevertheless answer the purpose of standards, that is they are invariable and can be reproduced at any time. On the other hand no reasonable end is served by throwing over the standards of saturation, intensity and hue (wave length) now becoming in general use. Therefore extensive measurements are now under way to establish formulas and tables for translating the readings of the apparatus into these standards.

[3] Schoen, Chem. Weekblad, *28*, 133; 1931.
[4] W. M. Scott, Textile World, *79*, 818; 1931.

CAMBRIDGE MEETING OF THE OPTICAL SOCIETY OF AMERICA

February 25–27, 1932
Joint Meeting with the American Physical Society

A joint meeting of the Optical Society of America and the American Physical Society was held in Cambridge, Massachusetts February 25 to 27, 1932.

The sessions on Thursday, under the auspices of the Optical Society of America were held at the Massachusetts Institute of Technology, and Friday and Saturday in Pierce Hall, Harvard University.

Notable features of this meeting were: (1) Joint conferences of the two societies on Thursday morning from 11:30 to 12:30 on "Electron Emission and Surface Conditions," "Perturbation Effects in Spectra," "Demonstration and Discussion of Mechanical Aids to the Solution of Equations," and "X-ray and Photomicrographic Studies in Metallurgy." (2) The joint symposium on "Properties of Matter" on Thursday afternoon at 2:30 followed by invited papers. (3) The evening lecture on Thursday at 8:15 on "Revealing the Universe through the Spectroscope" by Professor Henry Norris Russell. (4) On Thursday the members of the Societies and their guests were guests of the Massachusetts Institute of Technology at a luncheon at the Walker Memorial Building, and Friday they were guests of Harvard University at a luncheon given at Leverett House. (5) On Friday afternoon a joint symposium was held on "Electronic Devices and their Applications to Research with Special Reference to Radiation Sensitive Devices." (6) A dinner was given on Friday evening at 7:30 at the Parker House, Boston, presided over by E. C. Crittenden, President of the Optical Society of America at which time the Frederic Ives Medal, awarded at the Annual Meeting of the Optical Society of America, was presented to Professor Theodore Lyman in recognition of his distinguished contributions to the science of Optics. (7) A lecture Saturday morning at Harvard University by Professor P. W. Bridgman on "Anomalies in the Behavior of Solids under Pressure."

At the dinner Friday evening the members of the two Societies gave a unanimous vote of thanks to Harvard University and the Massachusetts Institute of Technology for the hospitality which had made the meeting possible.

AT THE JOINT SYMPOSIUM ON "PROPERTIES OF MATTER"
AT MASSACHUSETTS INSTITUTE OF TECHNOLOGY
THURSDAY AFTERNOON THE FOLLOWING
INVITED PAPERS WERE PRESENTED:

P. Debye — **University of Leipzig**
INTERFEROMETRIC MEASUREMENTS ON SINGLE MOLECULES

B. E. Warren — **Massachusetts Institute of Technology**
CRYSTAL STRUCTURE OF THE SILICATES

C. P. Smyth — **Princeton University**
MOLECULAR SHAPE AND ORIENTATION IN LIQUIDS

AT THE JOINT SYMPOSIUM ON "ELECTRONIC DEVICES AND THEIR APPLICATIONS TO RESEARCH WITH SPECIAL REFERENCE TO RADIATION SENSITIVE DEVICES" ON FRIDAY AFTERNOON THE FOLLOWING INVITED PAPERS WERE PRESENTED:

A. W. Hull — **General Electric Company**
ELECTRONIC DEVICES AS AIDS TO RESEARCH

N. P. Rashevsky — **Westinghouse Research Laboratory**
SOME PRACTICAL APPLICATIONS OF ELECTRONIC DEVICES

G. F. Metcalf — **General Electric Company**
SPECIAL TUBES FOR THE AMPLIFICATION OF SMALL VOLTAGES

Simultaneous Sessions of the American Physical Society and the Optical Society of America for the reading of contributed papers were held on Thursday Morning, Friday Morning and Saturday Morning. The author's abstracts of the papers presented before the Optical Society are appended.

Sessions of the Executive Council were held Wednesday afternoon at 2:30 p.m. and Wednesday evening at 7:45 p.m.

(Signed) L. B. Tuckerman,
Secretary.

MINUTES OF THE EXECUTIVE COUNCIL

Meeting at Boston, Massachusetts, February 24, 1932

Besides the transaction of routine business, the following items of general interest to members of the Society were taken up.

1. *Appointments approved by letter ballot.* Appointment of Committees and Representatives to other organizations which had been approved by letter ballot was announced:

Associate Editors:

K. K. Darrow, I. C. Gardner, G. R. Harrison, Loyd A. Jones, L. W. McKeehan, J. P. C. Southall.

Representatives in American Institute of Physics:

Paul D. Foote (Term expires Dec. 31, 1932), Loyd A. Jones (Term expires Dec. 31, 1933), F. K. Richtmyer (Term expires Dec. 31, 1934).

Representatives on Intersociety Color Council:

Loyd A. Jones appointed as additional representative.

The other members appointed and approved by letter ballot previous to the Council Meeting in October 1931 are:

Adelbert Ames, Jr., A. E. O. Munsell, A. H. Pfund, I. G. Priest (D. B. Judd, alternate), L. T. Troland, W. B. VanArsdel.

2. *Appointments by President.* With the approval of the Council the President appointed the following Committees:

Membership Committee:

T. R. Wilkins, Chairman, Frank Allen, Saul Dushman, H. W. Farwell, I. C. Gardner, L. Gilchrist, A. C. Hardy, George Harrison, H. E. Howe, L. R. Ingersoll, P. E. Klopsteg, H. F. Kurtz, E. O. Lawrence, H. M. Randall, J. B. Nathanson, M. Rea Paul, E. P. T. Tyndall.

Finance Committee:

Adolph Lomb, Chairman; Loyd A. Jones, F. K. Richtmyer, J. P. C. Southall.

Committee on Publications:

F. K. Richtmyer, Chairman; Paul D. Foote, Loyd A. Jones.

Committee on Meetings and Programs:

A. C. Hardy, Chairman; Herbert E. Ives, C. C. Kiess, Brian O'Brien.

Committee on Transfer Associate Members to Regular Members:

L. B. Tuckerman, Chairman; Adelbert Ames, Jr., J. A. Anderson, Raymond T. Birge, J. W. Buchta, Henry Crew, K. T. Compton, Heber D. Curtis, K. K. Darrow, J. S. Friedenwald, W. E. Forsythe, L. A. Grondahl, A. C. Hardy, Selig Hecht, Wm. F. Meggers, C. E. K. Mees, A. H. Pfund, W. B. Rayton, T. R. Wilkins.

Committee on Honorary Members:

George B. Pegram, Chairman; Henry Crew, W. B. Rayton.

Colorimetry Committee:

I. G. Priest, Chairman.

Auditors of Treasurer's Accounts:

J. P. C. Southall, C. L. Treleaven.

Auditors of Books of the Journal:

G. E. Grantham, H. E. Howe.

3. *Amendment to By-Laws:* The Secretary announced the following amendment to the By-Laws had been approved by letter ballot:

Moved that Article III—OFFICERS, of the By-Laws be amended by adding:

Section 4. Representatives and Delegates.—The President shall submit to the Council nominations for representatives and delegates to other organizations. Representatives and delegates shall be elected by the Council for such terms as their respective offices may require.

4. *Representatives to other organizations.* In accordance with the amendment to the By-Laws the following nominations by the President were approved by the Council:

Bureau of Standards Advisory Committee on International Electrical and Photometric Units and Standards:

F. K. Richtmyer (Indefinite term of office)

U. S. National Committee of the International Commission on Illumination:

Frank Benford (Term expires Dec. 31, 1934)

National Research Council, Division of Physical Sciences:

I. G. Priest (Term expires June 30, 1933)

Council of the American Association for the Advancement of Science:

H. E. Ives, P. I. Wold } (Indefinite term of office)

Intersociety Color Council:

Additional representative: I. H. Godlove.

5. *Transfer of the business office of the Journal.* The Council voted that:

The Executive Council of the Optical Society of America accepts the proposal of the American Institute of Physics that it take over the publication and printing of the Journal of the Optical Society of America and The Review of Scientific Instruments on July 1, 1932, for the remainder of the year 1932, and that the details of the arrangements, including editing, be referred to a special committee to consist of the President, the Secretary and the Committee on Publications, with power to act.

and concerning the policy to be followed after December 31, 1932, the Council voted that:

The Special Committee shall negotiate with the American Institute of Physics looking toward a unified policy of publications in physics, including a uniform format, and shall have power to make definite arrangements after consulting with the members of the Executive Council by mail.

6. *Charge for papers in the Journal.* The Council further voted that:

In conformity with the proposal of the American Institute of Physics, the Executive Council of the Optical Society of America agrees that beginning with July 1, 1932, a charge of two dollars ($2.00) per page will be made for articles published in the Journal of the Optical Society of America, this charge not to be an obligation of the author of the paper but of the institution supporting the research represented by the article.

7. *Place for 1932 Annual Meeting.* The Council also voted that:

The Annual Meeting of 1932 shall be held October 13, 14 and 15 at the Lake Placid Club provided satisfactory arrangements can be made with the Club management.

8. *Incorporation of the Society.* The Council voted that:

The Society shall incorporate in the City of Rochester, in the County of Monroe, in the State of New York.

It was noted that it would be necessary to secure a vote of the members of the Society authorizing a committee of the Society to take the necessary steps before the Society could incorporate. As soon as arrangements are made, ballots will be sent to the members.

9. *Intersociety Color Council.* The Council voted that:

The Secretary of the Optical Society is instructed to communicate with the President of the Intersociety Color Council indicating that the Optical Society of America will be a member of the Council.

(Signed) L. B. Tuckerman, Secretary

PROGRAM OF SESSIONS FOR CONTRIBUTED PAPERS

(TITLES AND ABSTRACTS OF PAPERS)

The abstracts were preprinted in the program subject to correction by the authors. Corrections that have been communicated to the secretary have been made in the abstracts reprinted below.

CONTRIBUTED PAPERS

Thursday Morning at 10 o'clock

Massachusetts Institute of Technology

Brian O'Brien **Institute of Optics, University of Rochester, N. Y.**

A NULL METHOD OF PHOTOGRAPHIC SPECTROPHOTOMETRY WITH A SPIRAL APERTURE SECTOR DISC

The addition of a projected intensity scale to the spiral aperture disk of Belin[1] previously reported,[2] while very satisfactory for photographic spectral energy measurement, does not permit spectrophotometric determinations of transmission or reflection except with a light source which can be held constant over an interval of several minutes. This limitation has been removed by substituting for the opaque intensity scale a series of spectra formed by light which has passed through or been reflected from the substance being measured, but has not passed through the spiral aperture disk. Thus a series of 22 identical spectra of light which has suffered absorption are alternated with 21 comparison spectra the average intensities of which are reduced in steps of $10^{0.1}$ between successive spectra by passage through the rotating spiral aperture disk. A density range from 0.0 to 2.0 is thus obtained which may be increased to 3.0 or higher by the addition of a neutral density such as a wire screen. It has been shown that reduction in average intensity by an intermittent exposure of this type is photographically equivalent to reduction in instantaneous intensity by the same amount, even for sectors giving a very high ratio of rest to flash period.[3]

The photographic plate is simultaneously exposed to both sets of spectra, so the light source used need not be held constant. The interposition of the two sets of spectra is accomplished either by means of a toothed metal mirror with metal comparison mirror, or by means of a special form of multiple Lummer Brodhun cube of quartz placed between the spectrograph slit and the spiral aperture disc, with a total reflecting quartz comparison prism. The new method is equal in accuracy to the usual form of sector photometer, but is far more rapid, since a complete absorption spectrum may be recorded with a single exposure.

[1] Belin, (Brit., J., Phot., p. 630 (Aug., 1906)).
[2] O'Brien (Phys. Rev., *33*, 640; 1929).
[3] O'Brien (Phys. Rev., *37*, 471; 1931).

Brian O'Brien **Institute of Optics, University of Rochester, N. Y.**

A PHOTOCELL FILTER COMBINATION WITH SPECTRAL SENSITIVITY APPROXIMATING THE VISIBILITY FUNCTION

The spectral sensitivity of a number of Weston Photronic cells has been determined from λ2500A to the long wave-length limit of sensitivity in the infrared. Eight cells of two slightly different types agree quite closely in spectral sensitivity throughout the visible spectrum, although the absolute sensitivities of some of the cells differ by a factor of 2. The mean spectral sensitivity curve has a maximum at λ5850A, but departs in form so widely from that of the eye (photopic) that errors of nearly 50% may be encountered in comparing lights of large color difference.

Using stable inorganic salts in a single solution, a light filter has been prepared which reduces the mean spectral sensitivity curve of the cell to a close approximation of the I. C. I. standard visibility function. While the differences between the sensitivity curve for the corrected cell and the visibility function are appreciable at certain wave lengths near the ends of the visible spectrum, the departures are so distributed about the visibility curve that the average sensitivity of the corrected cell agrees very closely with that of the eye throughout the entire spectrum. Infrared and ultraviolet sensitivities of the cell plus filter are quite negligible. Calculated response curves of the corrected cell and the eye to sources of color temperatures from 1900°K to 24,000°K, and to several sources emitting line spectra, show errors of less than $\pm\frac{1}{2}\%$ in the integrated response of the cell compared with that of the eye.

With suitable precautions taken to allow for the moderate fatigue effects and the slight departures from linearity of response exhibited by these cells, the combination of cell and filter is useful in the investigation of certain visual problems.

Arthur C. Hardy and O. W. Pineo **Mass. Institute of Technology**

THE APPLICABILITY OF PHOTOELECTRIC CELLS TO PHOTOMETRY AND COLORIMETRY

The various characteristics of photoelectric cells that make them unsuitable for direct methods of photometry and colorimetry have been discussed by Ives.* These characteristics have no effect on the precision of the proper type of null method. The present paper defines the conditions under which the photoelectric cell must be used in such a method, and contains also a description of an instrument that fulfills these conditions. The instrument employs two Rochon prisms and a Wollaston prism. One of the Rochon prisms is rotated at a high speed to produce the alternation of the two beams required by the null method. The other Rochon prism is used to control the relative intensities of the two beams like the rotating Nicol prism in a Martens photometer.

This instrument can be used for the direct comparison of two sources of the same spectral quality, or, in conjunction with a monochromator, as a spectrophotometer. The accuracy with which a single setting can be made is enormously greater than with visual instruments. When the azimuth of the Rochon prism is 45°, a setting can be made with no uncertainty to the nearest hundredth of a degree, which is the finest division that can be estimated on the tangent screw. This uncertainty corresponds to $\pm 0.035\%$ in the relative intensity of the two beams.

* Journal of the Optical Society of America, *21*, p. 541; 1931

C. E. Kenneth Mees **Eastman Kodak Company**

PHOTOGRAPHIC PLATES FOR USE IN SPECTROSCOPY AND ASTRONOMY II

The present paper is supplementary to that published in the Journal of the Optical Society for December, 1931. An extremely contrasty, slow emulsion has been found very suitable for

sensitizing, especially for the infrared portion of the spectrum and has been added to the basic emulsions classified in the previous paper. Several improvements have been made in emulsions of high green sensitivity. For general sensitizing up to the extreme red, the type of *F* sensitizing has now been modified to eliminate the gap in the blue-green. A new sensitizing method has been found to fill the gap previously existing between the visible spectrum and the Infra-Red Sensitive plates. Finally a sensitizer which has recently been obtained gives extraordinary sensitiveness to the extreme infrared, permitting the photography of spectra to beyond 11,000A.

The paper will be illustrated with lantern slides.

The present paper will appear in full in Journal of the Optical Society of America.

W. F. Meggers, C. C. Kiess and C. J. Humphreys **Bureau of Standards**

INFRARED SPECTRUM PHOTOGRAPHY

The well-known advantages of photographic methods of recording and investigating spectra have, for more than three decades, inspired attempts to extend the range of observation. For the infrared, it has been customary to sensitize ordinary photographic materials by bathing in solutions of sensitizing dyes, or in recent years by hypersensitizing so-called "infrared sensitive" emulsions. For more than a decade "dicyanine" held first place as the most useful dye for infrared photography; its sensitizing maximum occurs in the red at 7000A but with terrific exposures it has been known to respond to strong lines in the infrared between 9000A and 10,000A. The discovery of "neocyanine" in 1926 was hailed as a great advance because it imparted maximum sensitivity at 8100A, but its sensitizing curve is much steeper than that of dicyanine, and, therefore, has about the same limit in the infrared.

The recent discovery, in the Eastman Research Laboratories, of a new dye with sensitizing maximum at 9600A is now recognized as beginning a new era in infrared spectroscopy. Experimental plates kindly supplied by Dr. Mees have been tested at the Bureau of Standards in a preliminary survey of the infrared emission of 36 chemical elements (P, C, Si, Ti, Zr, V, Cb, Th, Cu, Ag, Zn, Cd, Hg, Cr, Mo, W, Ta, Mn, Re, Fe, Co, Ni, Na, K, Ca, Sr, Ba, Al, La, As, Bi, Sb, Kr, Xe, Yb, Lu). In nearly all cases many new lines have been discovered between 9000A and 11,200A.

These new photographic plates have two very remarkable properties:

(1) A very broad sensitizing band which ranges from about 8000A to 11,000A with maximum at 9600A.

(2) Extraordinary speed which is estimated at 100 to 1000 times that of the best materials heretofore available for the range 9000A to 10,000A.

These plates have been found to be perfectly satisfactory as regards sensitivity and freedom from excessive fog after five weeks in cold storage.

The present paper will appear in full in Bureau of Standards Journal of Research.

Friday Morning at 10 o'clock

Harvard University

CONTRIBUTED PAPERS

Deane B. Judd **Bureau of Standards**

NEAREST COLOR TEMPERATURE FOR STIMULI YIELDING NON-PLANCKIAN BUT NEARLY ACHROMATIC COLORS

Davis[1] has described a general empiric method by means of which any color may be correlated with some one Planckian color and has shown that the Planckian color obtained in

this way agrees well with that found experimentally to be the closest chromaticity match according to the rather meager data available. By making use of a previously derived coordinate system,[2] it is conveniently possible to deduce from the Priest-Brickwedde[3] data on minimum perceptible colorimetric purity the nearest color temperature for stimuli yielding non-Planckian but nearly achromatic colors. The method assumes (1) an observer characterized by the O. S. A. "excitation" curves[4] who (2) has a sensibility to change in colorimetric purity near zero purity essentially like that found by Priest for his own eye. Since in this method it is necessary to deduce from data applying to chromaticities near that of Abbot-Priest sunlight the chromaticity sensibility near other somewhat differing colors (Planckian colors from 4,000 to 8,000°K), the method may be described as one of extrapolation; but the rather considerable extent of the colors over which this extrapolation has been carried is justified by the fact that extrapolation all the way to spectrum colors (a gap of about 4 times this extent) results in errors not impossibly large.

Although the present method is equally as rapid as Davis' method and may be applied over the entire range of Planckian colors with about equal justification, there is little to be gained by changing to this method because within the range (4,000 to 8,000°K) for which this method is judged to be reliable it yields results which agree well with the Davis method and may be said to confirm that method and add materially to the confidence with which it may be used. Furthermore, the sort of discrepancies which are produced by extending the present method to the colors of the spectrum suggests another modification of the coordinate system within which it is possible that this method of extrapolation may be reliably extended to all colors.

[1] Davis, B. S. J. Research, *7*, 659–682; 1931.
[2] Judd, J.O.S.A. and R.S.I., *16*, 115; 1928; B. S. J. Research, *4*, 538; 1930.
[3] Priest and Brickwedde, J.O.S.A. and R.S.I., *13*, 306; 1926.
[4] Troland, J.O.S.A. and R.S.I., *6*, 548; 1922.

Deane B. Judd **Bureau of Standards**

INVESTIGATION OF SOURCES OF ERRORS IN COLOR TEMPERATURE DETERMINATIONS OF INCANDESCENT LAMPS

It is desirable to use a method of comparing incandescent lamps as to color temperature which is known to be free from constant errors. In an attempt to formulate such a procedure the following possible sources of error have been investigated:

(1) Change in the color temperature of the radiant energy emitted from various types of incandescent filament with direction.

(2) Change in color temperature of a lamp at constant applied voltage with heat insulation of housing.

(3) Change in observed color temperature due to long-period and short-period fluctuations in retinal sensitivity.

The first error sources have been found to introduce an uncertainty of about 3° for the type of filament used for standards of color temperature at 2848°K, but no measurable change was found for the squirrel-cage type of filament used for standards of color temperature at 2360°K. The error introduced by the second possible source was found to be negligible. But the error introduced by the third source—fluctuation in retinal sensitivity—is important because it makes a strict substitution method unattainable and because errors of this kind have been found to be as much as 10° at 2360°K.

I. H. Godlove **Munsell Research Laboratory**

STANDARDIZATION OF MUNSELL COLORS

Strongly contrasted with the physical method of spectrophotometry, with its advantages of precision, reproducibility, and freedom from the personal equation, is the ideal psychological

method, dealing with the three variables or attributes of color, which states directly "how a color looks." An ultimate ideal of colorimetry is the complete correlation or translation of these extreme methods. Intermediate between these are the colorimetric methods of trichromatic analysis, and of comparison with a system of reflection standards. The Munsell system can be considered as a quasi-psychological one, a convenient working approximation to the ideal; it has now been calibrated by trichromatic analysis by two or more observers, using the Guild colorimeter, and the results have been expressed in terms of the O. S. A. excitations. From spectral measurements, a tristimulus specification for a number of colors has also been computed on the same basis, and the two were found to agree well. Graphs of r vs. g and certain auxiliary graphs reveal for 400 colors relations between Munsell notation and the tristimulus specification which permits the intertranslation of the two. In turn, one can then translate to the other languages of color. There is also afforded a method of finding, at least approximately, a tristimulus specification of colors of other systems, as of Maerz and Paul's. In the measurements, the colorimetric differences between observers, the reproducibility of each, the deviations of the hues of individual colors from the hue average of all colors of a given hue-designation, and the differences between colorimetric measurements and computed spectrophotometric measurements, all roughly correspond in hue to about one step on the Jones hue scale and in saturation to a discrimination step of about one-half that size. Within the limitations of experimental error, and the departure of the Munsell charts from a psychological ideal, and with the assumption of a specific illumination and a specific observer, it is possible to pass from physical measurements to a psychological specification, and *vice versa* to a possible spectral distribution.

I. H. Godlove **Museum of Science and Industry of New York**

NOTE ON A COLOR BLINDNESS TEST OF MORE THAN 6500 PERSONS

Six thousand five hundred and thirty-nine persons attending the Color Exhibition of the Museum of Science and Industry of New York were tested for abnormalities of color vision by means of Ishihara charts. A form was used which permitted a record of the sex (70% were male), age (72% were below 30), and occupation. Ten male and one female completely color-blinds, no yellow-blue blinds, and 259 male and 24 female red-green blinds, were found. That is, 5.9% of the men, and 1.3% of the women, were abnormal. These figures are higher than those usually published. They include a number of persons using color in their daily work, as interior decorators, and even painters who claimed to be artists. The scientific value of the results for any population is in question since there was no compulsion to take the test, so that (1) sensitive persons having some inkling of a defect may have avoided the test, and (2) persons suspicious of a color defect may have purposely come in for examination.

Arthur C. Hardy and O. W. Pineo **Mass. Institute of Technology**

THE COMPUTATION OF TRICHROMATIC EXCITATION VALUES BY THE SELECTED ORDINATE METHOD

The three excitation values that describe the sensation evoked by a given stimulus are obtained by integrating throughout the spectrum the product of each of the three excitation functions by a function representing the energy distribution of the stimulus. These functions are of such a nature that either a graphical or a point-by-point method of integration must be adopted. In the latter case, the customary procedure is to select points in the spectrum that are separated by intervals of 10 mμ, to multiply the energy at each point by the corresponding values of the three excitation functions in turn, and to sum the results. In the method proposed in this paper, the laborious multiplication is avoided by selecting the points in such a manner that a mere addition of the values of the energy at these points gives the trichromatic values directly.

A mechanical integrating device to be used in conjunction with an automatic spectrophotometer for the computation of excitation values has been described by one of the present authors.* The selected ordinate method permits this device to be vastly simplified because, with the necessity of multiplication avoided, the device reduces to an adding machine.

* Journal of the Optical Society of America, *18*, p. 96 (1929).

Irwin G. Priest — **U. S. Representative on Colorimetry Committee of International Commission on Illumination**

REPORT ON THE WORK OF THE COLORIMETRY COMMITTEE OF THE INTERNATIONAL COMMISSION ON ILLUMINATION AND THE ACTIONS TAKEN AT THE MEETING IN CAMBRIDGE, ENGLAND, SEPTEMBER, 1931. PRESENTED BY D. B. JUDD IN THE ABSENCE OF MR. PRIEST

The purposes of this report are: (1) to give some account of the activities of the Colorimetry Committee of the I. C. I., and particularly the American participation in these activities since the I. C. I. meeting at Saranac Inn, September, 1928; (2) to communicate the resolutions proposed by the Colorimetry Committee at the I. C. I. meeting in Cambridge, England, September, 1931.

The resolutions proposed at Cambridge, England, have been adopted tentatively by the I. C. I. and will be considered as definitely adopted unless protests are received at the central office of the I. C. I. by March 18, 1932. These resolutions deal with standards and conventions in colorimetrics, particularly with: (1) the specification of the hypothetical observer to be considered as standard in color specifications; (2) the specification of illuminants to be considered as standard in specifying the colors of non-self-luminous objects; (3) the mode of illumination, angle of regard, and substance to be considered as standard in specifying the colors of samples viewed by reflected light; (4) conventions as to reference stimuli, units, and systems of trilinear coordinates to be used in specifying color stimuli. Copies of these resolutions, as received from the U. S. National Committee, may be obtained by addressing Dr. D. B. Judd, Bureau of Standards, Washington, D. C. The U. S. representatives[1] on the committee will be glad to receive any comments of any Americans who may be interested in such standardization.

[1] I. G. Priest was U. S. Representative for the period of the committee's work ended with the meeting in Cambridge, England, September, 1931. I. G. Priest and D. B. Judd have been appointed U. S. Representatives on the committee for the period ending with the meeting which will be held in Germany in 1934.

REPORT FROM REPRESENTATIVES OF THE OPTICAL SOCIETY IN THE INTERSOCIETY COLOR COUNCIL AND DISCUSSION OF THE REPORT, LED BY LOYD A. JONES, ACTING CHAIRMAN OF THE OPTICAL SOCIETY OF AMERICA REPRESENTATIVES

Mr. Jones reported briefly on the First Annual Meeting of the Intersociety Color Council, which had been held in New York City December 29, 1931. The Optical Society of America had been represented at the meeting by:

Deane B. Judd (alternate for I. G. Priest), Loyd A. Jones, A. E. O. Munsell.

The Articles of Organization and Procedure, prepared by the Executive Committee had been unanimously adopted and the following officers elected for the coming year:

E. N. Gathercoal, Chairman; Loyd A. Jones, Vice Chairman; M. Rea Paul, Secretary; A. E. O. Munsell, Treasurer.

Additional members of the Executive Committee:

Charles Bittinger, Mrs. Margaret H. Rorke, Walter M. Scott.

The following Committees were appointed to further the work of the Council:

Committee on Measurement and Specification of Color:

I. H. Godlove, Chairman.

Committee on Color Names:
A. E. O. Munsell, Chairman.

Committee on Color Problems:
Mrs. Margaret H. Rorke, Chairman.

Committee to Investigate Requirements in Industry for Individuals Trained in Color:
I. H. Godlove, Chairman.

Following Mr. Jones' report, Mr. Godlove, Mr. Munsell and Mr. Paul discussed briefly the problems which lay before the Council, emphasizing the purpose of the Council to act chiefly as a correlating and coordinating agency between the widely different organizations which had such diverse interests in color problems.

Saturday Morning at 9:30 o'clock
Harvard University

CONTRIBUTED PAPERS

Harlan T. Stetson — **Perkins Observatory, Ohio Wesleyan University**

OPTICAL TESTS OF THE 69-INCH PERKINS OBSERVATORY REFLECTOR

The large disk of glass, cast by the Bureau of Standards in May, 1927, has emerged into the completed paraboloidal reflector which has just recently been placed in the Perkins Observatory telescope at Delaware, Ohio.

Before removal of the glass to Delaware an extensive series of optical tests were made in the testing laboratory of the J. W. Fecker Company, in Pittsburgh, where the optical surface of the mirror was ground and figured. The final tests were carried on over a period of five days, Dr. J. S. Plaskett, Director of the Dominion Astrophysical Observatory, and Dr. I. C. Gardner of the Bureau of Standards cooperating. Some 3000 settings were made by the knife edge test at the center of curvature. Excellent agreement was found among the different observers. Computed results show the mean focal length of the mirror to be $7^{m}614$ (24 ft. $11\frac{3}{4}$ in.), and the departures from this value for different zones of varying radii to be not over 1/10 of a millimeter as would be tested in parallel light reflected to a focus.

Tests were also made by the Hartmann method and a comparison made between the results of the two methods. The provisional value of the Hartmann criterion for the mirror from laboratory tests is placed at 0.064.

I. C. Gardner — **Bureau of Standards**

AN ATTACHMENT FOR PRODUCING APPROXIMATELY SPHERICAL SURFACES OF SMALL CURVATURE ON A LATHE

This device was particularly designed for the production of the concave or convex surfaces of lens grinding tools although it is well adapted for the production of such surfaces for any purpose. The attachment is mounted on the lathe in the place of the compound slide rest and the tool is guided by a simple linkage for which no curved templates of any sort are required. Settings for any desired radius of curvature within the range of the instrument are readily made by altering the position of one member of the linkage. This attachment is intended for the productions of surfaces of which the radius of curvature is too great to permit the convenient use of a tool mounted on the end of a radius rod. The attachment as constructed permits discs under 300 mm in diameter to be faced and any radius of curvature greater than 1000 mm may be obtained with a deviation from the true spherical surface which does not exceed 0.1 mm.

E. C. Crittenden **Bureau of Standards**

PROGRESS TOWARD INTERNATIONAL AGREEMENT ON PHOTOMETRIC STANDARDS

While nominally all recent photometric data are based on one or the other of two units, the international candle and the Hefnerkerze, which have the simple ratio of 10 to 9 in magnitude, actually this degree of uniformity is not obtained in practice. In passing from the older standards to modern lamps operating at higher temperatures different countries have used different methods, and those which have used the ordinary photometer for this purpose have gotten results discordant among themselves. The establishment of a uniform system of photometry applicable to lights of all colors has been delayed by differences of opinion regarding the validity of methods of observation as well as by the lack of a suitable basic or primary standard.

Two international organizations are attempting to develop agreements on photometric standards. These are the official International Committee on Weights and Measures, working through its Advisory Committee on Electricity and Photometry, and the International Commission on Illumination, which has recently appointed a Special Committee on Photometry including representatives from six countries. This report summarizes the present status of proposals before the two organizations.

In brief, with few exceptions, authorities agree that a primary standard is to be sought in some form of black-body radiator; the majority think it is expedient to use the black body at the freezing point of platinum as the primary standard, and the specific form known as the Waidner-Burgess standard[1] is strongly supported by definite experimental results as well as theoretical advantages.

On methods of observation no general agreement has been reached, but the national laboratories of France, Germany, Great Britain and the United States have agreed to bring their standards operating at various temperatures into concordance by using blue filters calibrated by spectrophotometric measurements. Transmissions of the filters for integral light will be calculated from the spectra data by means of the "visibility factors" accepted by the International Commission on Illumination.[2]

[1] Wensel, Roeser, Barbrow and Caldwell, Bur. Stds. Jour. Res. *6*, pp. 1103–1117; 1931. (Res. Paper No. 325.)
[2] Trans. Ill. Eng. Soc. 25, p. 731, October, 1930. See also Gibson and Tyndall, Bur. Stds. Sci. Papers *19*, pp.131–191; 1923–4 (S.P. No. 475.)

H. T. Wensel **Bureau of Standards**

APPLICATIONS TO PHOTOMETRY OF A BLACK BODY IMMERSED IN FREEZING IRIDIUM

The action of the Colorimetry Committee of the International Commission on Illumination in adopting as standards three illuminants all of which call for the operation of a gas-filled tungsten lamp at a color temperature of 2848°K, either with or without specified filters, makes a reliable standard of color temperature in the neighborhood of this temperature indispensable. A black body immersed in freezing iridium offers the possibility of providing such a standard at 2726°K which has one big advantage over lamp standards in that it is reproducible. In addition the color temperature of this standard, being the same as its true temperature, has been determined with an accuracy, within 3°K, far greater than that with which the color temperature of a lamp around 2848°K can be independently determined.

Furthermore, the action of the International Committee of Weights and Measures in charging its Advisory Committee on Electricity to take up the question of photometric standards in view of the possibility of securing a reproducible standard in some form of black body radiator makes the suitability of a black body immersed in freezing iridium of particular interest. While the Waidner-Burgess standard, black body immersed in freezing platinum, has every advantage over the above black body in freezing iridium, the latter may be of im-

portance in providing one more method for stepping up from carbon to tungsten lamps. The relative brightness, candles per cm², of black bodies at the platinum and iridium points can be calculated from their temperatures.

At the last meeting a report was made of work in which the relative brightness calculated from the temperatures was compared with that obtained by comparing the black bodies with carbon and tungsten lamps matching the respective black bodies in color. The agreement, 1.5%, was within experimental error. More recently a direct measurement of the relative brightness for red light of these two standards has been made yielding relative values for the two temperatures. This permitted a more accurate calculation of the relative brightness and resulted in a somewhat better agreement, slightly less than 1%.

17. T. C. Fry **Bell Telephone Laboratories, Inc.**

PLANE WAVES OF LIGHT III: ABSORPTION BY METALS

A continuation of the detailed study of plane light waves which was begun in two earlier papers with the same general title (J.O.S.A. *15;* 1927, pp. 137–161 and *16;* 1928, pp. 1–25). The present study discusses not only the absorbing power of bulk material, but also, and more particularly, the absorption which takes place in a thin layer just below the reflecting surface, or in a thin layer of one kind of material resting on a block of another kind. This "lamellar absorption" varies with wave length, plane of polarization and angle of incidence in a way quite different from the "bulk absorption" of ordinary optics. Furthermore, the lamellar absorption of a layer resting on a block of another material may be very different from the bulk properties either of the material of the layer or of the material of the block.

The results may be of importance in the study of photoemission. All previous attempts to correlate photoemissivity with optical absorption seem to have considered only the bulk absorbing power of the photo-active material. If any such correlation exists we would rather expect it to be with the lamellar absorbing power instead, since it is improbable that electrons can escape unless released very near the surface.

The study is made entirely on the basis of the classical Maxwell theory of continuous media.

The present paper will be submitted to J.O.S.A

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA
THE REVIEW OF SCIENTIFIC INSTRUMENTS

Statement of Cash Receipts and Disbursements

For the Period from January 1, 1931 to December 31, 1931

Cash on Hand, January 1, 1931		$ 441.46
Receipts:		
In account with the Optical Society of America (12 issues at $400.00 each)		4,800.00
In account with Scientific Apparatus Makers of America Advertising		7,872.00
Miscellaneous Receipts:		
Subscriptions	$ 3,943.05	
Sale of back numbers	465.40	
Overpayments	29.53	
Transfer from interest account	3,000.00	7,437.98
Total		$20,551.44
Disbursements:		
Printer	$14,374.54	
Salaries and office expenses	5,756.09	
Miscellaneous	163.78	$20,294.41
Cash on hand, December 31, 1931		257.03
Total		$20,551.44

The above is a correct statement of accounts to the best of my knowledge and belief.

(Signed) F. K. Richtmyer, Business Manager

We, the undersigned, duly appointed to audit the account of the Journal of the Optical Society of America and The Review of Scientific Instruments at the close of business, December 31, 1931, hereby certify that we have examined the statement of receipts, and disbursements, the vouchers, check book and bank book and find them to agree with each other. We, therefore, certify that the above statement is a true statement of the accounts of the Journal of the Optical Society of America and The Review of Scientific Instruments as of December 31, 1931.

(Signed) Harley Howe
G. E. Grantham
Auditors

REVIEWS

S.M.P.E. COMMITTEE REPORTS RECOMMENDATIONS FOR 16MM FILM STANDARDS

The sub-committee of the Standards Committee of the Society of Motion Picture Engineers has reported its recommendations for 16 mm film standards, together with complete layouts for two types of 16 mm film. These recommendations are now up for approval and validation by the Standards Committee and the general Society.

Before recommendations could be made it was necessary for the committee to make a thorough study of all problems in connection with 16 mm film. At present two types of 16 mm film are under consideration by the industry—the first being a film with one row of perforations and a sound track on the other side of the film—the second being the present type, embodying two sets of sprocket perforations. After detailed study of these two systems, the committee has drawn up standards for each type of film, but has also made its recommendation favoring one system.

Other problems calling for examination were:—(1) the relative screen illumination available, and the requisite amount of magnification of the projected picture; (2) whether the projector aperture should be fixed or adjustable; (3) whether or not the shape of the projected picture will conform to the shape of existing screens; (4) the amount of space available for the sound track in either case; (5) whether duplicate negatives can be used for printing, or not; (6) whether the film can be turned over in the gate, or not; (7) whether it will be possible to run the film in existing machines; (8) whether direct optical reduction may be used for printing; and (9) the possibility of causing modulation of the recorded sound at the perforation frequency due to the insufficient space between the perforations and the sound track.

The work of the sub-committee was directed by Mr. J. L. Spence; others cooperating in the work were Messrs. V. B. Sease, P. H. Evans, L. A. Jones, H. T. Jermaine, H. Griffin, R. P. May, T. E. Shea, L. A. Elmer, and H. G. Tasker. The entire Standards Committee, which is finally to act upon the recommendations of the sub-committee, is headed by Mr. M. C. Batsel, chairman.

Photo-Elasticity. Coker and Filon. Cambridge: at The University Press: New York: The Macmillan Company. 720 pages, $17.50.

Photoelastic methods of stress analysis have provided engineers with a tool for the solution of many problems which have heretofore been exceedingly difficult of analysis. This book offers to science and industry a thoroughly comprehensive consideration of the subject from the standpoints of both theory and practice.

The first two chapters consider the elements of the theories of optics and elasticity in relation to photoelasticity. The phenomena of propagation of light in crystals, interference, reflection, refraction, polarization, etc., are explained by methods of the classical electromagnetic theory. The third chapter sketches the history of photoelasticity with particular emphasis being placed on those phases of development which most clearly illustrate the principal aspects of the subject.

The remaining chapters deal with the numerous problems arising from the application of photoelastic theory to industry. Consideration in detail is given to problems in plane stress and strain involving both straight and curved boundaries, beam flexure, distribution of stress about holes, cracks, notches and discontinuities and stress distribution due to bending, contact pressures, etc. Of particular interest to investigators in the field of properties of materials is the chapter dealing with stress conditions in tensile impact, compression, and shear tests.

The final chapter deals with structures and machines and a consideration of the many factors involved in the application of photoelasticity to design problems. The principles of dynamic similarity are developed and utilized for the study of structures and machines by the use of models. The examples are well chosen to illustrate the possible applications of photoelastic methods to the solution of problems not readily susceptible to analysis by other means.

The book is unusually complete in its treatment of the subject and gives adequate consideration both to the theoretical principles and practical applications. The authors have been intimately connected with the development of photoelasticity since its inception and the book offers convincing evidence of the thoroughness of their study and experimentation. It will be found exceedingly instructive and valuable by those interested in either the theoretical or practical aspects of photoelasticity.

E. N. KEMLER AND B. B. WESCOTT

Atombau und Spetrallinien, Vol. I. By Arnold Sommerfeld, Professor of Theoretical Physics at the University of Munich. viii+734 pages. Friedr. Vieweg and Son. Braunscheweig. Paper, RM 29; cloth, RM 31.60.

In bringing out the fifth edition of this well-known work, Professor Sommerfeld has found it necessary to divide the material into two volumes. This first volume is essentially a revision, with some abbreviations and other alterations, of the fourth edition. Volume II, to appear presently, is to be an expansion of the Wellen-Mechanischer Ergänzungsband.

Previous editions of "Atombau und Spektrallinien" are so well known and have been so widely used that little need be said in the present volume. As the author remarks in the preface, a fundamental knowledge of the older theories of atomic structure and radiation are prerequisite to an under-

standing of the newer. It is in large part the purpose of this first volume to present the empirical facts of spectra and the older theories based thereon to prepare the way for volume II.

There are nine chapters, viz: Preliminary Facts; The Hydrogen Spectrum; The Natural System of the Elements; X-ray Spectra; Theory of Fine Structure; Polarization and Intensity of Spectral Lines; The General Laws of Spectral Series; The Complex Structure of Series Terms; and Band Spectra. There are some thirteen appendices.

Physicists throughout the world are grateful to Professor Sommerfeld for bringing out from time to time a new edition of this very important book in which is presented in easily accessible and well digested form the kaleidoscopic developments in the field of atomic structure and radiation.

F. K. RICHTMYER

Strahlenoptik. Dr. M. Herzberger. Vol 35 of Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen. viii+196 pages. Published by Julius Springer, Berlin. 1931.

Optische Abbildung, Einführung in die Wellen- und Beugungstheorie optischer Systeme. Dr. Johannes Picht. Vol 84 of Die Wissenschaft. x+243 pages. Friedr. Vieweg & Sohn Akt.-Ges., Braunschweig. 1931.

These two books complement each other so well in subject matter that it seems advisable to treat them in a single review. The first, by Herzberger, bases the derivation of the equations of geometric optics upon the stationary value of the light path. The equations for tracing rays through an optical system, for Gaussian imagery, for non-orthogonal imagery and the development of the eikonal are treated. The author then reverts to Gaussian imagery, discusses the theory of the action of the diaphragms of an optical system as developed by Abbe and v. Rohr, and derives the expressions for the Seidel aberrations. All of this material is given in a condensed but lucid form and the presentation possesses elegance and unity because of its derivation from a single principle. Furthermore the use of the stationary light path and the introduction of the eikonal serves to relate the subject matter more closely to the wave theory of light than does the older geometric presentation. However it should not be overlooked that in all cases the final equations deal with rays as, in fact, the author has indicated by the title of his book.

The second book by Picht, is a most interesting treatise in which the character of imagery produced by an optical system is considered from the standpoint of the wave theory of light. The first fifty pages are devoted to a brief formulation of the electromagnetic theory of light embracing refraction, reflection, Huyghen's principle and Kirchoff's more rigorous formulation of it. Spherical waves and cylindrical waves, in the absence of aberra-

tion, are next considered and expressions derived for the intensity and phase at points in the neighborhood of the image, including, not only those lying in the image plane but also those in front of and behind this plane. Wave fronts free from geometric aberrations, but non-homogeneous are next considered. A non-homogeneous wave front is one for which the amplitude, over the wave-front, is not constant. Certain important special cases of non-homogeneity are assumed and the manner in which the diffraction pattern differs from that of the corresponding homogeneous wave is clearly set forth.

In the last half of the book a method is given by which the diffraction pattern corresponding to a wave front deformed from the spherical form by the presence of typical geometric aberrations can be found and the method is applied to estimate the effects of the different aberrations. Throughout the book an abundance of well selected graphs assist greatly to acquaint the reader with the significance of the different mathematical results which are obtained.

At present, the design of an optical system must proceed in two steps. By the methods of geometric optics the courses of selected rays must be followed and the values of the geometric aberrations obtained. For the final interpretation of the geometric aberrations and a determination of the quality of imagery to be expected it is necessary to consider the geometric aberrations in terms of the wave theory. These two volumes do not give detailed methods of computation such as are presented, for example, by Conrady, and applications of the deductions to specific optical instruments are not made. They do present, however, in remarkably compact form, a complete discussion of the underlying fundamentals of optical design.

I. C. GARDNER

Motion Picture Museum—The Society of Motion Picture Engineers has established in the Los Angeles Museum a museum containing several thousand objects which show the evolution of the motion picture industry. This motion picture museum is managed by the Museum Committee of the Society under the chairmanship of Mr. E. Theisen.

The exhibits are arranged as far as possible in a chronological order arranging them in such manner so the pioneers represented are given credit for the influence they exerted upon subsequent screen history. The labels on the exhibits are accurate, the authority for them being obtained from the Patent Office records and by referring to the pioneers making them. They are so labeled with a view in mind of a possible use of the exhibit for reference.

Among the exhibits represented are:—Muybridge with his "Horse in Motion" experiments for Leland Stanford, various models of projectors including Edison, Edison Exhibition Model, Pathe, Amet, Edengraph, Motio-

graph, Kinema-Kolor, and etc. There is a collection of some three thousand stills, hand bills, posters of the nickelodeon theatres, props used in pictures, miniatures, and photographs of prominent people.

One exhibit that represents about six year's work is a collection of authentic specimens of film made by the various pioneers. These specimens are a frame or two in length and are bound between glass for preservation. These actual specimens of film show in a concise manner the evolution of the industry. Accompanying each specimen is a historical notation on the pioneer making the specimen.

There are about twelve hundred specimens in this collection and among them can be seen films of any width from four millimeters up to four inches representing various attempts to establish other than the thirty-five millimeter width. There are some two hundred different color attempts recorded. This collection includes besides color, and the various width attempts, sound, third dimension, processes, and outstanding pictures; in fact, everything that is made on film including the first piece of film made on the celluloid supplied by George Eastman to Edison in 1889, and transparent paper used prior to the advent of celluloid.

Mrs. Alice Herbert, the widow of the late Thomas H. Ince, has deposited at the Museum, a collection of ninety-four albums containing the complete collection of synopses of the pictures directed by Ince. Accompanying these snyopses is a complete set of stills of the pictures. This dates back to 1929 when Mr. Ince first contacted with motion pictures with Biograph.

Another interesting collection on exhibit was made by David Horsley in 1920. This is a collection of famous stars of 1915 to 1920. They are hand-colored transparencies. Mr. Horsley was an outstanding independent producer. He formed the Centaur and Nester Film Companies, also assisting in the formation of Universal.

The Los Angeles Museum in which the motion picture museum is housed is maintained by the county and is open seven days a week. It occupies a pretentious building in Exhibition Park. There are about five hundred visitors daily.